

$K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay correlating with ϵ_K in high-scale SUSY

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Abstract

We have studied the contribution of the high-scale SUSY to the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ processes by correlating with the CP violating parameter ϵ_K . Taking account of the recent LHC results for the Higgs discovery and the SUSY searches, we consider the high-scale SUSY at the 10 – 50TeV scale in the framework of the non-minimal squark (slepton) flavor mixing. The Z penguin mediated the chargino dominates the SUSY contribution for these decays. At the 10TeV scale of the SUSY, the chargino contribution can enhance the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in eight times compared with the SM predictions whereas the predicted branching ratio $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ increases up to three times of the SM one. The gluino box diagram dominates the SUSY contribution of ϵ_K up to 30%. If the down-squark mixing is neglected compared with the up-squark mixing, the Z penguin mediated the chargino dominates both SUSY contributions of $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and ϵ_K . Then, it is found a correlation between them, but the chargino contribution to ϵ_K is at most 3%. Even if the SUSY scale is 50TeV, the chargino process still enhances the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ from the SM prediction in the factor two, and ϵ_K is deviated from the SM prediction in $\mathcal{O}(10\%)$. We also discuss the chargino contribution to $K_L \rightarrow \pi^0 e^+ e^-$ process.

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1 Introduction

The K meson physics have provided important informations in the indirect search for New Physics (NP). Especially, the rare decay processes $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ are known as the clean one theoretically [1, 2]. Therefore, these both processes have been considered to be one of the powerful probes of NP [3]–[14] whereas these decay widths are bounded by so called the Grossman-Nir bound for the NP [15, 16].

The $K_L \rightarrow \pi^0 \nu \bar{\nu}$ process is the CP violating one and provides the direct measurement of the CP violating phase in the Cabibbo-Kobayashi-Maskawa (CKM) matrix [17, 18]. In addition, the CP conserving process $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is also the physical quantity related with the unitarity triangle (UT). On the other hand, the CP violating parameter ϵ_K , which is induced by the $K^0 - \bar{K}^0$ mixing, also constrains the height of the UT. Hence these measured variables give us the information of the UT fit as well as the CP violating quantity $\sin 2\phi_1$ induced by the $B^0 - \bar{B}^0$ mixing. Furthermore, the $K \rightarrow \pi \nu \bar{\nu}$ processes are expected to open the NP window in the CP violating flavor structure.

The $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay processes are governed by the Z penguin diagram in the Standard Model (SM) [19], which predicts

$$\begin{aligned} BR(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{SM}} &= (2.43^{+0.40}_{-0.37} \pm 0.06) \times 10^{-11}, \\ BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{SM}} &= (7.81^{+0.80}_{-0.71} \pm 0.29) \times 10^{-11}. \end{aligned} \quad (1)$$

In the estimation of the branching ratio of $K \rightarrow \pi \nu \bar{\nu}$, the hadronic matrix elements can be extracted with the isospin symmetry relation [20, 21]. These processes are theoretically clean because the long-distance contributions are small [12], and then the theoretical uncertainty is estimated below several percent. On the other hand, ϵ_K has the different flavor mixing structure from these processes since it is induced by the box diagram of $K^0 - \bar{K}^0$ mixing. Therefore, the NP is expected to appear in both $K \rightarrow \pi \nu \bar{\nu}$ and ϵ_K with different magnitudes.

On the experimental side, the upper bound of the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is given by the KEK E391a experiment [22]. The branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ measured by the BNL E787 and E949 experiments is consistent with the SM prediction [23];

$$\begin{aligned} BR(K_L \rightarrow \pi^0 \nu \bar{\nu})_{\text{exp}} &< 2.6 \times 10^{-8} \text{ (90\%C.L.)}, \\ BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})_{\text{exp}} &= (1.73^{+1.15}_{-1.05}) \times 10^{-10}. \end{aligned} \quad (2)$$

At present, the J-PARC KOTO experiment is an in-flight measurement of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ approaching to the SM predicted precision [24, 25], while the CERN NA62 experiment [26] studies the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ process.

On the theoretical side, the supersymmetry (SUSY) is one of the most attractive candidates for the NP. However, the SUSY signals have not been observed yet, and then the recent searches for new particles at the LHC give us important constraints for the SUSY. Since the lower bounds of masses of the SUSY particles increase gradually, the squark and the gluino masses are supposed to be at the higher scale than 1 TeV [27, 28, 29]. Moreover, the SUSY models have been seriously constrained by the Higgs discovery, in which the Higgs mass is 126 GeV [30, 31].

These facts suggest a class of SUSY models with heavy sfermions. If the squark and slepton masses are expected to be $\mathcal{O}(10 - 100)$ TeV, the lightest Higgs mass can be pushed

up to 126 GeV, whereas all SUSY particles can be out of the reach of the LHC experiment. Therefore, the indirect search of the SUSY particles becomes important in the low energy flavor physics [32, 33, 34].

So far, the effects of SUSY on the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ processes have been studied in the framework of the Minimal Supersymmetric Standard Model (MSSM) with the minimal flavor violation (MFV) scenario intensively [8, 10]. Since the SUSY mass scale is pushed up more than 1 TeV region at present, the effect of the MSSM with MFV is expected to be very small. These processes are also discussed in the framework of the general SUSY model [9, 35, 36, 37, 38, 39, 40] at the $\mathcal{O}(500)$ GeV scale.

We have studied the SUSY contribution to the CP violation of the B meson and ϵ_K induced by the $K^0 - \bar{K}^0$ mixing under the relevant SUSY particle spectrum constrained by the observed Higgs mass [34]. Then, it is found that the SUSY contribution could be up to 40% in the observed ϵ_K , on the other hand, it is minor in the CP violation of the B meson at the high scale of 10 – 50 TeV. Therefore, in this paper, we investigate the high-scale SUSY contribution to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ by correlating with ϵ_K in the framework of the mass eigenstate of the SUSY particles, which is consistent with the updated experimental situations like the direct SUSY searches and the Higgs discovery, with the non-minimal squark (slepton) flavor mixing.

Our paper is organized as follows. Sec.2 gives the basic framework of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and ϵ_K in the SM and the MSSM. In Sec.3, we present the setup of the high-scale SUSY. In Sec.4, we discuss our numerical results. Sec.4 is devoted to the summary. The SUSY mass spectra and the Z penguin amplitude mediated the chargino are given in Appendices A and B, respectively.

2 Basic framework

In this section, we present the basic formulae for the $K \rightarrow \pi \nu \bar{\nu}$ decay and the CP violating parameter ϵ_K , which correspond to $|\Delta S| = 1$ and $|\Delta S| = 2$ processes, respectively. The $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ processes are clean ones theoretically since the hadronic matrix elements can be extracted including isospin breaking corrections by taking the ratio to the leading semileptonic decay of $K^+ \rightarrow \pi^0 e^+ \nu$. Moreover, the long-distance contributions to these rare decays are negligibly small. Therefore, the accurate measurements of these decay processes provide the crucial tests of the SM. Especially, the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ process is purely the CP violating one, which can reveal the source of the CP violating phase.

On the other hand, the CP violating parameter ϵ_K is measured with enough accuracy. The major theoretical ambiguity comes from the hadronic matrix element factor \hat{B}_K . The recent lattice calculations give us the reliable value for \hat{B}_K [41, 42]. The more accurate estimate of the SM contribution enables us to search the NP such a SUSY because we know the accurate observed value of ϵ_K . Actually, the non-negligible SUSY contribution has been expected in ϵ_K at the scale of $\mathcal{O}(100)$ TeV [32, 33, 34]. Consequently, it is required to examine the high-scale SUSY contribution in $K \rightarrow \pi \nu \bar{\nu}$ by correlating with ϵ_K .

2.1 Basic framework : $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$

2.1.1 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in the SM

Let us start with discussing the framework of the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ processes in the SM [1]. The effective Hamiltonian for $K \rightarrow \pi \nu \bar{\nu}$ in the SM is given:

$$\mathcal{H}_{\text{eff}}^{\text{SM}} = \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} \sum_{i=e,\mu,\tau} [V_{cs}^* V_{cd} X_c + V_{ts}^* V_{td} X_t] (\bar{s}_L \gamma^\mu d_L) (\bar{\nu}_L^i \gamma_\mu \nu_L^i) + \text{H.c.}, \quad (3)$$

which is induced by the box and the Z penguin mediated the W boson. The dominant box contribution is derived by the top-quark exchange, on the other hand, the charm-quark exchange contributes to the Z penguin process as well as the top-quark one. The up-quark contribution is negligible due to its small mass. So, the loop function X_c denotes the charm-quark contribution of the Z penguin, and X_t is the sum of the top-quark exchanges of the box diagram and the Z penguin in Eq.(3).

Let us define the function F as follows:

$$F = V_{cs}^* V_{cd} X_c + V_{ts}^* V_{td} X_t. \quad (4)$$

The branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is given in terms of F . Taking the ratio of it to the branching ratio of $K^+ \rightarrow \pi^0 e^+ \bar{\nu}$, which is the tree level process, we obtain a simple form:

$$\frac{BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})}{BR(K^+ \rightarrow \pi^0 e^+ \bar{\nu})} = \frac{2}{|V_{us}|^2} \left(\frac{\alpha}{2\pi \sin^2 \theta_W} \right)^2 \sum_{i=e,\mu,\tau} |F|^2. \quad (5)$$

The $K^+ \rightarrow \pi^0 e^+ \bar{\nu}$ decay is precisely measured as $BR(K^+ \rightarrow \pi^0 e^+ \bar{\nu})_{\text{exp}} = (5.07 \pm 0.04) \times 10^{-2}$ [43], and its hadronic matrix element is related to the one of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ with the isospin symmetry:

$$\langle \pi^0 | (\bar{d}_L \gamma^\mu s_L) | \bar{K}^0 \rangle = \langle \pi^0 | (\bar{s}_L \gamma^\mu u_L) | K^+ \rangle, \quad (6)$$

$$\langle \pi^+ | (\bar{s}_L \gamma^\mu d_L) | K^+ \rangle = \sqrt{2} \langle \pi^0 | (\bar{s}_L \gamma^\mu u_L) | K^+ \rangle, \quad (7)$$

where the coefficients are determined by the Clebsch-Gordan coefficient. By using this relation, the hadronic matrix element has been removed in Eq.(5).

Now the branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ is expressed as follows:

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = 3\kappa \cdot r_{K^+} |F|^2, \quad (8)$$

$$\kappa = \frac{2}{|V_{us}|^2} \left(\frac{\alpha}{2\pi \sin^2 \theta_W} \right)^2 BR(K^+ \rightarrow \pi^0 e^+ \bar{\nu}), \quad (9)$$

where r_{K^+} is the isospin breaking correction between $K^+ \rightarrow \pi^0 e^+ \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ [20, 21], and the factor 3 comes from the sum of three neutrino flavors. It is noticed that the branching ratio for $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ depends on both the real and imaginary part of F .

For the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decay, the $K^0 - \bar{K}^0$ mixing should be taken account, and one obtains

$$\begin{aligned}
A(K_L \rightarrow \pi^0 \nu \bar{\nu}) &= \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} (\bar{\nu}_L^i \gamma_\mu \nu_L^i) \langle \pi^0 | [F(\bar{s}_L \gamma_\mu d_L) + F^*(\bar{d}_L \gamma_\mu s_L)] | K_L \rangle \\
&= \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} (\bar{\nu}_L^i \gamma_\mu \nu_L^i) \frac{1}{\sqrt{2}} [F(1 + \bar{\epsilon}) \langle \pi^0 | (\bar{s}_L \gamma_\mu d_L) | K^0 \rangle + F^*(1 - \bar{\epsilon}) \langle \pi^0 | (\bar{d}_L \gamma_\mu s_L) | \bar{K}^0 \rangle] \\
&= \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} (\bar{\nu}_L^i \gamma_\mu \nu_L^i) \frac{1}{\sqrt{2}} [F(1 + \bar{\epsilon}) - F^*(1 - \bar{\epsilon})] \langle \pi^0 | (\bar{d}_L \gamma_\mu s_L) | K^0 \rangle \\
&\simeq \frac{G_F}{\sqrt{2}} \frac{2\alpha}{\pi \sin^2 \theta_W} (\bar{\nu}_L^i \gamma_\mu \nu_L^i) \frac{1}{\sqrt{2}} 2\text{Im}F \langle \pi^0 | (\bar{d}_L \gamma_\mu s_L) | K^0 \rangle.
\end{aligned} \tag{10}$$

In the step of the first line going to the second line in (10), we use

$$|K_L\rangle = \frac{1}{\sqrt{2}} [(1 + \bar{\epsilon})|K^0\rangle + (1 - \bar{\epsilon})|\bar{K}^0\rangle], \tag{11}$$

and then, after using the CP transition relation in the second line,

$$\text{CP}|K^0\rangle = -|\bar{K}^0\rangle, \quad \text{C}|K^0\rangle = |\bar{K}^0\rangle, \tag{12}$$

$$\langle \pi^0 | (\bar{d}_L \gamma_\mu s_L) | \bar{K}^0 \rangle = -\langle \pi^0 | (\bar{s}_L \gamma_\mu d_L) | K^0 \rangle, \tag{13}$$

we obtain the equation in the third line. In the final line, we neglect the CP violation in $K^0 - \bar{K}^0$ mixing, $\bar{\epsilon}$, due to its smallness $|\bar{\epsilon}| \sim 10^{-3}$.

Taking the ratio between the branching ratio of $K^+ \rightarrow \pi^0 e^+ \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$, we have the simple form:

$$\frac{BR(K_L \rightarrow \pi^0 \nu \bar{\nu})}{BR(K^+ \rightarrow \pi^0 e^+ \bar{\nu})} = \frac{2}{|V_{us}|^2} \left(\frac{\alpha}{2\pi \sin^2 \theta_W} \right)^2 \frac{\tau(K_L)}{\tau(K^+)} \sum_{i=e,\mu,\tau} (\text{Im}F)^2. \tag{14}$$

Therefore, the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is given as follows:

$$BR(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 3\kappa \cdot \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)} (\text{Im}F)^2, \tag{15}$$

where r_{K_L} and r_{K^+} denote the isospin breaking effect [20, 21]. It is remarked that the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ depends on the imaginary part of F . Since the charm-quark contribution is negligible due to the small imaginary part of $V_{cs}^* V_{cd}$, it is enough to consider only the top-quark exchange in this decay.

In the SM, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ are related to the UT fit. We write down the branching ratio in terms of the Wolfenstein parameters. Since $\text{Re}F$ and $\text{Im}F$ are given as

$$\text{Re}F = -\lambda X_c - A^2 \lambda^5 (1 - \rho) X_t, \quad \text{Im}F = A^2 \lambda^5 \eta X_t, \tag{16}$$

we can express the branching ratio of these decays as

$$\begin{aligned}
BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) &= 3\kappa \cdot r_{K^+} [(\text{Re}F)^2 + (\text{Im}F)^2] \\
&= 3\kappa \cdot r_{K^+} \cdot A^4 \lambda^{10} X_t^2 \left[(\bar{\rho} - \rho^0)^2 + \bar{\eta}^2 \right],
\end{aligned} \tag{17}$$

where

$$\rho_0 = 1 + \frac{X_c}{A^2 \lambda^4 X_t} , \quad (18)$$

and

$$\begin{aligned} BR(K_L \rightarrow \pi^0 \nu \bar{\nu}) &= 3\kappa \cdot \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)} (\text{Im}F)^2 \\ &= 3\kappa \cdot \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)} \cdot A^4 \lambda^{10} X(x_t)^2 \eta^2 . \end{aligned} \quad (19)$$

It is noticed that $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ in Eq(17) is approximately a circle centered at $\bar{\rho} = \rho_0 \simeq 1.2$, $\bar{\eta} = 0$ on the $\bar{\rho}$ - $\bar{\eta}$ plane. On the other hand, $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ in Eq(19) just depends on η and it can determine the height of the UT directly. In this way, the precise measurements of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ become crucial tests for the SM.

Before going to discuss the SUSY formulation, we present the general bound between $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$, so called the Grossman-Nir bound [15]. As seen from above formulations, since the two processes are determined by the imaginary part and the absolute value of the same coupling, the model independent bound is obtained:

$$BR(K_L \rightarrow \pi^0 \nu \bar{\nu}) < \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)} \cdot BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) \lesssim 4.4 \times BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) , \quad (20)$$

where we use the isospin symmetry $A(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \sqrt{2}A(K^0 \rightarrow \pi^0 \nu \bar{\nu})$. This bound must be satisfied for any NP [15, 16].

2.1.2 $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in the MSSM

The effective Hamiltonian in Eq.(3) is modified due to new box diagrams and penguin diagrams induced by SUSY particles. Then, the effective Lagrangian is given as

$$\mathcal{L}_{\text{eff}} = \sum_{i,j=e,\mu,\tau} [C_{\text{VLL}}^{ij} (\bar{s}_L \gamma^\mu d_L) + C_{\text{VRL}}^{ij} (\bar{s}_R \gamma^\mu d_R)] (\bar{\nu}_L^i \gamma_\mu \nu_L^j) + \text{H.c.} , \quad (21)$$

where i and j are the index of the flavor of the neutrino final state. Here, $C_{\text{VLL,VRL}}^{ij}$ is the sum of the box contribution and the Z penguin one:

$$\begin{aligned} C_{\text{VLL}}^{ij} &= -B_{\text{VLL}}^{21ij} - \frac{\alpha_2}{4\pi} Q_{ZL}^{(\nu)} P_{ZL}^{21} \delta^{ij} , \\ C_{\text{VRL}}^{ij} &= -B_{\text{VRL}}^{21ij} - \frac{\alpha_2}{4\pi} Q_{ZL}^{(\nu)} P_{ZR}^{21} \delta^{ij} , \end{aligned} \quad (22)$$

where the weak neutral-current coupling $Q_{ZL}^{(\nu)} = 1/2$, and $B_{\text{VL(R)L}}^{21ij}$ and $P_{ZL(R)}^{21}$ denote the box contribution and the Z penguin contribution, respectively. The V , L and R denote the vector coupling, the left-handed one and the right-handed one, respectively. In addition to the W

boson contribution, there are the gluino \tilde{g} , the chargino χ^\pm and the neutralino χ^0 mediated ones¹. We write each contribution as follows:

$$\begin{aligned}
B_{\text{VLL}}^{sdij} &= B_{\text{VLL}}^{sdij}(W) + B_{\text{VLL}}^{sdij}(\chi^\pm) + B_{\text{VLL}}^{sdij}(\chi^0), \\
B_{\text{VRL}}^{sdij} &= B_{\text{VRL}}^{sdij}(\chi^\pm) + B_{\text{VRL}}^{sdij}(\chi^0), \\
P_{\text{ZL}}^{sd} &= P_{\text{ZL}}^{sd}(W) + P_{\text{ZL}}^{sd}(H^\pm) + P_{\text{ZL}}^{sd}(\tilde{g}) + P_{\text{ZL}}^{sd}(\chi^\pm) + P_{\text{ZL}}^{sd}(\chi^0), \\
P_{\text{ZR}}^{sd} &= P_{\text{ZR}}^{sd}(\tilde{g}) + P_{\text{ZR}}^{sd}(\chi^\pm) + P_{\text{ZR}}^{sd}(\chi^0),
\end{aligned} \tag{23}$$

where (i, j) denotes the neutrinos of final state. Explicit expressions are given in Ref.[44]. It is well known that the most dominant contribution comes from the Z penguin mediated chargino for the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ decays [12].

The branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ and $K_L \rightarrow \pi^0 \nu \bar{\nu}$ are obtained by replacing internal effect F in Eqs. (8) and (15) to $C_{\text{VLL}}^{ij} + C_{\text{VRL}}^{ij}$ as follows:

$$BR(K^+ \rightarrow \pi^+ \nu \bar{\nu}) = \kappa \cdot r_{K^+} \sum_{i=e,\mu,\tau} |C_{\text{VLL}}^{ij} + C_{\text{VRL}}^{ij}|^2, \tag{24}$$

$$BR(K_L \rightarrow \pi^0 \nu \bar{\nu}) = \kappa \cdot \frac{r_{K_L}}{r_{K^+}} \frac{\tau(K_L)}{\tau(K^+)} \sum_{i=e,\mu,\tau} |\text{Im}(C_{\text{VLL}}^{ij} + C_{\text{VRL}}^{ij})|^2. \tag{25}$$

2.2 ϵ_K in the MSSM

It is well known that the CP violating parameter ϵ_K induced by the $K^0 - \bar{K}^0$ oscillation gives us one of the most serious constraint to the NP. The general expression for ϵ_K is given as

$$\epsilon_K = e^{i\phi_\epsilon} \sin \phi_\epsilon \left(\frac{\text{Im}(M_{12}^K)}{\Delta M_K} + \xi \right), \quad \xi = \frac{\text{Im}A_0}{\text{Re}A_0}, \tag{26}$$

where A_0 is the 0-isospin amplitude in the $K \rightarrow \pi\pi$ decay, and M_{12}^K is the dispersive part of the $K^0 - \bar{K}^0$ oscillations, and ΔM_K is the mass difference of the neutral K meson. The effects of $\xi \neq 0$ and $\phi_\epsilon < \pi/4$ were estimated by Buras and Guadagnoli [45]. In the SM, the off-diagonal mixing amplitude M_{12}^K is obtained as

$$\begin{aligned}
M_K^{12} &= \langle K^0 | \mathcal{H}_{\Delta S=2} | \bar{K}^0 \rangle \\
&= \frac{4}{3} \left(\frac{G_F}{4\pi} \right)^2 M_W^2 \hat{B}_K F_K^2 M_K \left[\eta_{cc}(V_{cs}V_{cd}^*)^2 S(x_c) + \eta_{tt}(V_{ts}V_{td}^*)^2 S(x_t) \right. \\
&\quad \left. + 2\eta_{ct}(V_{cs}V_{cd}^*)(V_{ts}V_{td}^*) S(x_c, x_t) \right],
\end{aligned} \tag{27}$$

where $S(x)$ denotes the SM one-loop functions [46], and $\eta_{cc,tt,ct}$ are the QCD corrections [45]. Recent lattice calculations give us the precise determination of the \hat{B}_K parameter [41, 42].

Once taking account of the NP effect, the expression of M_{12}^K is modified. In the case of the SUSY, new contributions to the box diagrams are given by the gluino \tilde{g} , the charged

¹The wino-higgsino mixing is tiny in our mass spectrum.

Higgs H^\pm , the chargino χ^\pm and the neutralino χ^0 exchanges:

$$\begin{aligned} M_{12}^K &= M_{12}^{K,\text{SM}} + M_{12}^{K,\text{SUSY}} \\ &= M_{12}^K(W) + M_{12}^K(H^\pm) + M_{12}^K(\chi^\pm) + M_{12}^K(\chi^0) + M_{12}^K(\tilde{g}) + M_{12}^K(\chi^0 \tilde{g}). \end{aligned}$$

The explicit formula has been presented in Ref. [44].

3 Setup of the squark flavor mixing

We present the setup of our calculation in the framework of the high-scale SUSY. Recent LHC results for the SUSY search may suggest the high-scale SUSY, $\mathcal{O}(10 - 1000)$ TeV [32, 33, 34, 47] since the lower bounds of the gluino mass and squark masses exceed 1 TeV. Taking account of these recent results, we consider the possibility of the high-scale SUSY at 10, 50 TeV, in which the $K \rightarrow \pi \nu \bar{\nu}$ decays and ϵ_K are discussed..

Another important experimental result should be mentioned is the Higgs discovery. The Higgs mass $m_H \simeq 126$ GeV gives effect to the SUSY mass spectrum. In general, there are two possibility to get Higgs mass value, one is the heavy stop around 10 TeV, and the another is the large $X_t = A_0 - \mu \cot \beta$ given by the A-term. In the case that the SUSY scale is 10 to 50 TeV, we have already obtained the SUSY mass spectra which realize the Higgs mass at the electroweak scale with Renormalization Group Equation (RGE) running in previous work [34]. We use this numerical result for the SUSY particle mass spectrum. In this study, the 1st and 2nd squark are almost degenerated due to the assumption of the universal soft masses. On the other hand, the 3rd squark mass obtains the large contribution from the RGE running due to the large Yukawa coupling of the top-quark. Therefore, the mixing between 1st and 2nd is negligible, and it is taken account in the subsequent discussion for squark flavor mixing. The SUSY spectra at 10 and 50 TeV are given in Appendix A.

Once the SUSY mass spectrum is fixed, we can calculate the left-right mixing angle θ^q , which is defined as

$$\theta^d \simeq \frac{m_b(A_0 - \mu \tan \beta)}{m_{b_L}^2 - m_{b_R}^2}, \quad \theta^u \simeq \frac{m_t(A_0 - \mu \cot \beta)}{m_{t_L}^2 - m_{t_R}^2}. \quad (28)$$

In the case of the SUSY scale to be 10 and 50 TeV, the left-right mixing angles of squarks and sleptons are very small as ($\theta^d \sim 0.0062$, $\theta^u \sim 0.0024$, $\theta^e \sim 0.014$) and ($\theta^d \sim 0.0009$, $\theta^u \sim 0.0007$, $\theta^e \sim 0.005$), respectively.

The SUSY brings the new flavor mixing through the quark-squark-gaugino couplings and the lepton-slepton-gaugino ones. The 6×6 squark mass matrix $M_{\tilde{q}}^2$ in the super-CKM basis turns to the mass eigenstate basis by diagonalizing with rotation matrix $\Gamma^{(q)}$ as

$$m_{\tilde{q}}^2 = \Gamma^{(q)} M_{\tilde{q}}^2 \Gamma^{(q)\dagger}, \quad (29)$$

where $\Gamma^{(q)}$ is the 6×6 unitary matrix, and we decompose it into the 3×6 matrices as

$\Gamma^{(q)} = (\Gamma_L^{(q)}, \Gamma_R^{(q)})^T$ in the following expressions:

$$\Gamma_L^{(q)} = \begin{pmatrix} c_{13}^{qL} & 0 & s_{13}^{qL} e^{-i\phi_{13}^{qL}} c_{\theta^q} & 0 & 0 & -s_{13}^{qL} e^{-i\phi_{13}^{qL}} s_{\theta^q} e^{i\phi^q} \\ -s_{23}^{qL} s_{13}^{qL} e^{i(\phi_{13}^{qL} - \phi_{23}^{qL})} & c_{23}^{qL} & s_{23}^{qL} c_{13}^{qL} e^{-i\phi_{23}^{qL}} c_{\theta^q} & 0 & 0 & -s_{23}^{qL} c_{13}^{qL} e^{-i\phi_{23}^{qL}} s_{\theta^q} e^{i\phi^q} \\ -s_{13}^{qL} c_{23}^{qL} e^{i\phi_{13}^{qL}} & -s_{23}^{qL} e^{i\phi_{23}^{qL}} & c_{13}^{qL} c_{23}^{qL} c_{\theta^q} & 0 & 0 & -c_{13}^{qL} c_{23}^{qL} s_{\theta^q} e^{i\phi^q} \end{pmatrix},$$

$$\Gamma_R^{(q)} = \begin{pmatrix} 0 & 0 & s_{13}^{qR} s_{\theta^q} e^{-i\phi_{13}^{qR}} e^{-i\phi^q} & c_{13}^{qR} & 0 & s_{13}^{qR} e^{-i\phi_{13}^{qR}} c_{\theta^q} \\ 0 & 0 & s_{23}^{qR} c_{13}^{qR} s_{\theta^q} e^{-i\phi_{23}^{qR}} e^{-i\phi^q} & -s_{13}^{qR} s_{23}^{qR} e^{i(\phi_{13}^{qR} - \phi_{23}^{qR})} & c_{23}^{qR} & s_{23}^{qR} c_{13}^{qR} e^{-i\phi_{23}^{qR}} c_{\theta^q} \\ 0 & 0 & c_{13}^{qR} c_{23}^{qR} s_{\theta^q} e^{-i\phi^q} & -s_{13}^{qR} c_{23}^{qR} e^{i\phi_{13}^{qR}} & -s_{23}^{qR} e^{i\phi_{23}^{qR}} & c_{13}^{qR} c_{23}^{qR} c_{\theta^q} \end{pmatrix}, \quad (30)$$

where we use abbreviations $c_{ij}^{qL,qR} = \cos \theta_{ij}^{qL,qR}$, $s_{ij}^{qL,qR} = \sin \theta_{ij}^{qL,qR}$, $c_{\theta^q} = \cos \theta^q$ and $s_{\theta^q} = \sin \theta^q$. It is remarked that we take $s_{12}^{qL,qR} = 0$ due to the degenerate squark masses of the 1st and the 2nd families as noted in Appendix A. The angle θ^q is the left-right mixing angle between \tilde{q}_L and \tilde{q}_R , and they are calculable as mentioned above. Then, there are free mixing parameters $\theta_{ij}^{qL,qR}$ and $\phi_{ij}^{qL,qR}$. For simplicity, we assume $s_{ij}^{qL} = s_{ij}^{qR}$. On the other hand, we scatter ϕ_{ij}^{qL} and ϕ_{ij}^{qR} in the $0 \sim 2\pi$ range independently. It should be noted that the mixing angles $s_{ij}^{qL(R)}$ have not been constrained by the experimental data of B, D and K mesons in the framework of the high-scale SUSY [34].

For the lepton sector, the mixing matrices $\Gamma_{L(R)}^{(\ell)}$ have the same structure as the quark one in the charged-lepton flavors, however, there is only the left-handed $\Gamma_L^{(\nu)}$ in neutrinos.

As well known, the charged Higgs and the chargino contributions dominate the $K \rightarrow \pi \nu \bar{\nu}$ processes [12]. Since the SUSY scale is high in our scheme, the charged Higgs are heavy, $\mathcal{O}(10\text{TeV})$, so the charged Higgs contribution is suppressed in our framework. On the other hand, the dominant SUSY contribution to ϵ_K comes from the gluino box diagram if the flavor mixing angles of the down-squark and the up-squark are comparable. In addition, the chargino box diagram is also non-negligible. Consequently, we will discuss the both cases in which the down-squark mixing angles $s_{ij}^{dL(R)}$ are negligible small and are comparable to the up-squark mixing angles $s_{ij}^{uL(R)}$. We scan the phases of Eq.(30) for up-squarks, down-squarks, charged-sleptons and sneutrinos in the region of $0 \sim 2\pi$ independently.

In our framework, the $K \rightarrow \pi \nu \bar{\nu}$ processes are dominated by the Z penguin mediated the chargino exchange, $P_{ZL}^{sd}(\chi^\pm)$ in Eq.(23), which are occurred through the $\tilde{t}_L s_L(d_L)\chi^\pm$ and $\tilde{t}_R s_L(d_L)\chi^\pm$ interactions, respectively. In our basis, the relevant mixing is given by

$$(\Gamma_{CL}^{(d)})^{\alpha q} \equiv (\Gamma_L^{(u)} V_{\text{CKM}})_I^q (U_+)_1^\alpha + \frac{1}{g_2} (\Gamma_R^{(u)} \hat{f}_U V_{\text{CKM}})_I^q (U_+)_2^\alpha, \quad (31)$$

where $q = s, d$, $I = 1 - 6$ for up-squarks, and $\alpha = 1, 2$ for charginos. The V_{CKM} is the CKM matrix, and U_+ is the 2×2 unitary matrix which diagonalize $M_C^\dagger M_C$, where M_C is the 2×2 chargino mass matrix. The \hat{f}_U denotes the yukawa coupling defined by $\hat{f}_U v \sin \beta = \text{diag}(m_u, m_c, m_t)$. Therefore, the combinations of mixing angles and phases in Eq.(30), $c_{13}^{qL} s_{13}^{qL} s_{23}^{qL} e^{i(\phi_{13}^{qL} - \phi_{23}^{qL})}$ and $c_{13}^{qR} s_{13}^{qR} s_{23}^{qR} e^{i(\phi_{13}^{qR} - \phi_{23}^{qR})}$ are important for our numerical analysis in the next section. We show the formula for $P_{ZL}^{sd}(\chi^\pm)$ in Appendix B.

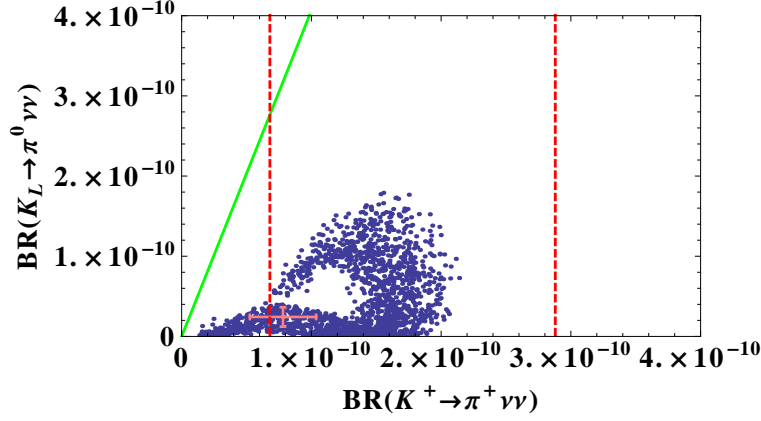


Figure 1: The predicted $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ at the SUSY scale of 10 TeV with the mixing angle of $s^u = s^d = 0.1$. The pink cross denotes the SM predictions. The red dashed lines are the 1σ experimental bounds for $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. The green slanting line shows the Grossman-Nir bound.

4 Numerical analysis

Let us discuss the high-scale SUSY contribution to the $K \rightarrow \pi \nu \bar{\nu}$ processes by correlating with ϵ_K [13]. At present, we cannot confirm whether the SM prediction ϵ_K^{SM} is in agreement with the experimental value ϵ_K^{exp} because there remains the theoretical uncertainty with an order of a few ten percent. However, the theoretical uncertainties of ϵ_K are expected to be reduced significantly in the near future. Actually, the lattice calculations of \hat{B}_K will be improved significantly [41, 42], whereas $|V_{cb}|$ and the CKM phase γ will be measured more precisely in Belle-II. Therefore, we will be able to test the correlation between $K \rightarrow \pi \nu \bar{\nu}$ and ϵ_K .

In our previous work, we have examined the sensitivity of the high-scale SUSY with 10 and 50 TeV to ϵ_K . It is found that the SUSY contribution to ϵ_K is allowed up to 40%. We begin to discuss the SUSY contribution at the 10 TeV scale. The present uncertainties in the SM prediction for ϵ_K are due to the CKM elements V_{cb} , $\bar{\rho}$ and $\bar{\eta}$, and the \hat{B}_K parameter. We take the CKM parameters V_{cb} , $\bar{\rho}$ and $\bar{\eta}$ at the 90 % C.L. of the experimental data:

$$|V_{cb}| = (41.1 \pm 1.3) \times 10^{-3}, \quad \bar{\rho} = 0.117 \pm 0.021, \quad \bar{\eta} = 0.353 \pm 0.013. \quad (32)$$

For the \hat{B}_K parameter, the recent result of the lattice calculations is given as [41, 42];

$$\hat{B}_K = 0.766 \pm 0.010, \quad (33)$$

which is used with the error-bar of 90% C.L. in our calculation.

In the beginning, we show the numerical results at the SUSY scale of 10 TeV. Fig.1 shows the predictions on the $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ vs. $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ plane, where phase parameters are constrained by the observed $|\epsilon_K|$ with the experimental error-bar of 90% C.L. Here, we fix the mixing parameters in Eq.(30) by taking the common value $s_{i3}^{uL} = s_{i3}^{uR} = s^u = 0.1$ ($i = 1, 2$)

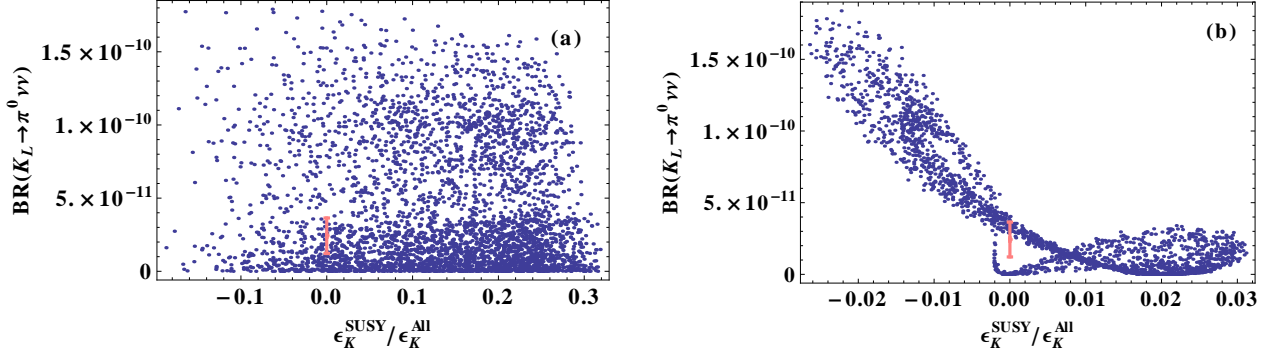


Figure 2: The predicted $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus the SUSY contribution ratio of ϵ_K at the SUSY scale of 10 TeV in the case of (a) $s^u = s^d = 0.1$ and (b) $s^u = 0.1$, $s^d = 0$. The pink short line denotes the SM prediction with the error-bar of 90% C.L.

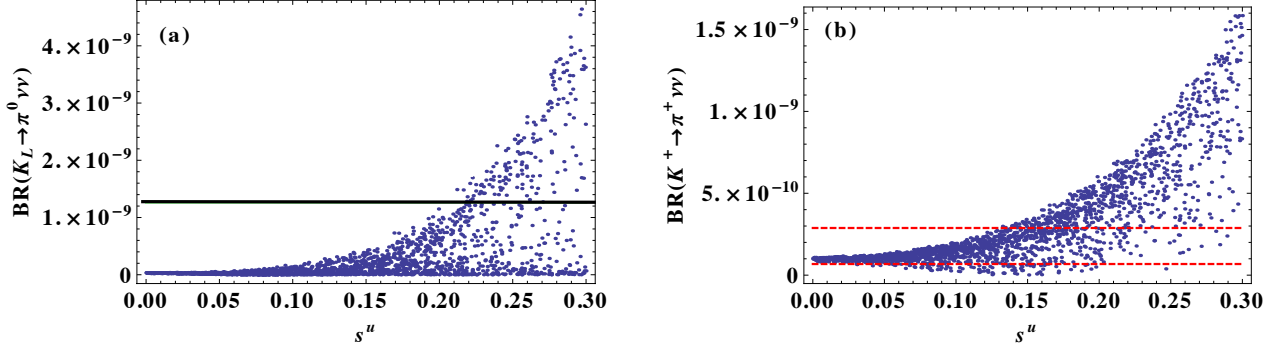


Figure 3: The predicted s^u dependence of (a) $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and (b) $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ at the SUSY scale of 10 TeV, where s^d is scanned in the region of $0 \sim 0.3$ independent of s^u . The red dashed lines denote the 1σ experimental bounds for $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. The black line corresponds to the Grossman-Nir bound together with the experimental upper bound of $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ with 3σ .

and $s_{i3}^{dL} = s_{i3}^{dR} = s^d = 0.1$ ($i = 1, 2$) for the up-quark and the down-quark sectors, respectively. The Z penguin mediated chargino dominates the SUSY contribution to these branching ratios.

The SUSY contributions can enhance the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in eight times compared with the SM predictions in Eq.(1), 1.8×10^{-10} although it is much smaller than the Grossman-Nir bound. On the other hand, the predicted $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ increases up to three times, 2.1×10^{-10} . It is also noticed that the predicted region of $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is reduced to much smaller than 10^{-11} due to the cancellation between the SM and SUSY contributions. The $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ could be reduced to 1.3×10^{-11} .

We discuss the correlation between ϵ_K and $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ in Fig. 2, in which (a) $s^u = s^d = 0.1$ and (b) $s^u = 0.1$, $s^d = 0$. The transverse axis denotes the SUSY contribution in $|\epsilon_K|$. If the down-squark mixing s^d is comparable to the up-squark mixing s^u , there is no correlation between them as seen in Fig. 2(a), where the Z penguin mediated chargino dominates the SUSY contribution of $K_L \rightarrow \pi^0 \nu \bar{\nu}$, and the gluino box diagram dominates the

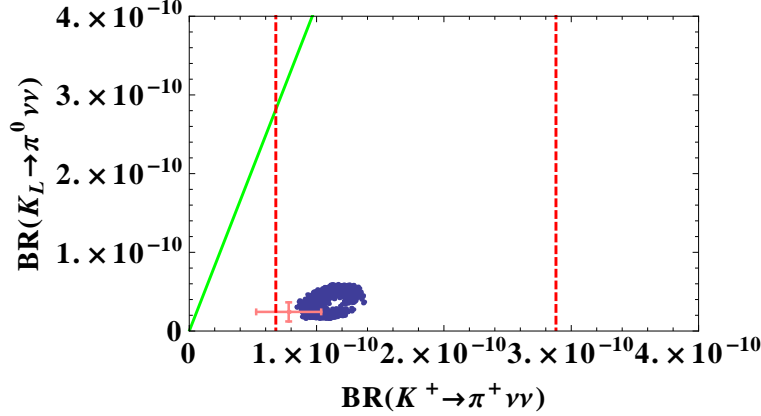


Figure 4: The predicted $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ at the SUSY scale of 50 TeV with the mixing angle of $s^u = s^d = 0.3$. The pink cross denotes the SM predictions. The red dashed lines are the 1σ experimental values for $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$. The green slanting line shows the Grossman-Nir bound [15].

SUSY contribution of ϵ_K . The gluino contribution of 30% is possible in ϵ_K .

On the other hand, if the down-squark mixing s^d is tiny compared with the up-squark mixing s^u , the Z penguin mediated chargino dominates both SUSY contributions of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and ϵ_K . Then, it is found a correlation between them as seen in Fig. 2(b), where the chargino contribution to ϵ_K is at most 3%. This correlation is due to the difference of the phase structure between the penguin diagram and the box diagram of the chargino.

In conclusion, ϵ_K could be deviated from the SM prediction in $\mathcal{O}(10\%)$ due to the gluino box diagram, whereas the Z penguin mediated chargino could enhance the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ from the SM prediction.

Next, in order to see the mixing angle s^u dependence of the branching ratios, we plot the predicted regions on $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ vs. s^u and $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ vs. s^u planes taking $s^u = 0 \sim 0.3$ in Fig.3 (a) and (b). We scan s^d in the region of $0 \sim 0.3$ independent of s^u although the gluino contribution is much suppressed compared with the chargino one. In this plot, the SUSY contribution to ϵ_K is free ($0 - 40\%$), but the experimental constraint of $|\epsilon_K|$ with the error-bar of 90%C.L. is taken account. We show the upper bound given by the Grossman-Nir bound together with the experimental upper bound of $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ with 3σ by the black line, at which the predicted $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ should be cut. Namely, the observed upper bound of $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ gives the constraint for the predicted $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ at s^u larger than 0.2. The precise experimental measurement of $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ will lower the predicted upper bound of $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$.

Let us discuss the case of the SUSY scale of 50 TeV. Fig. 4 shows the predictions on the $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ plane at the SUSY scale of 50 TeV, where the mixing angle is fixed at $s^u = s^d = 0.3$. Although the predicted region is reduced considerably comparing to the case of the 10 TeV scale in Fig. 1, the predicted branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is enhanced in two times from the SM prediction, and the branching ratio of $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ could be enhanced from the SM prediction in three times.

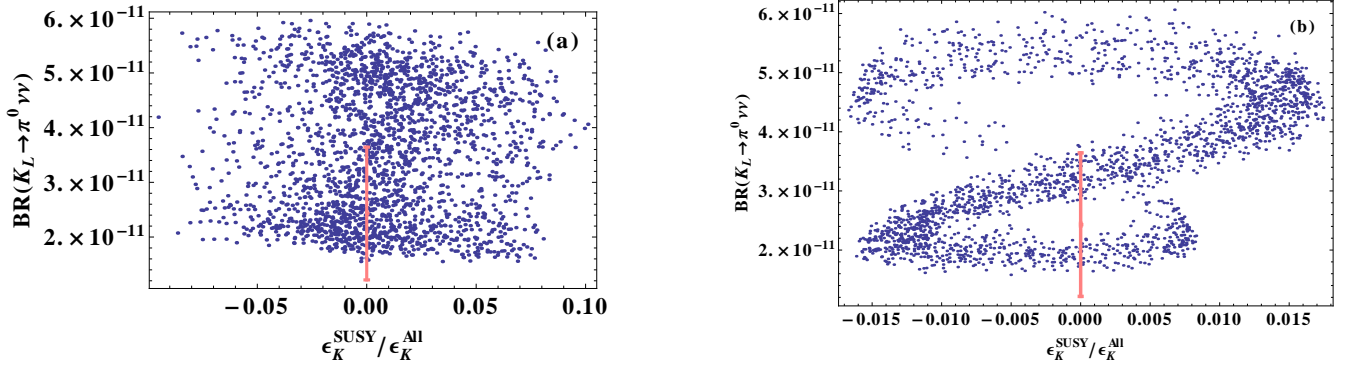


Figure 5: The predicted $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ versus the SUSY contribution ratio of ϵ_K at the SUSY scale of 50 TeV in the case of (a) $s^u = s^d = 0.3$ and (b) $s^u = 0.3$, $s^d = 0$. The pink short line denotes the SM prediction with the error-bar of 90%C.L.

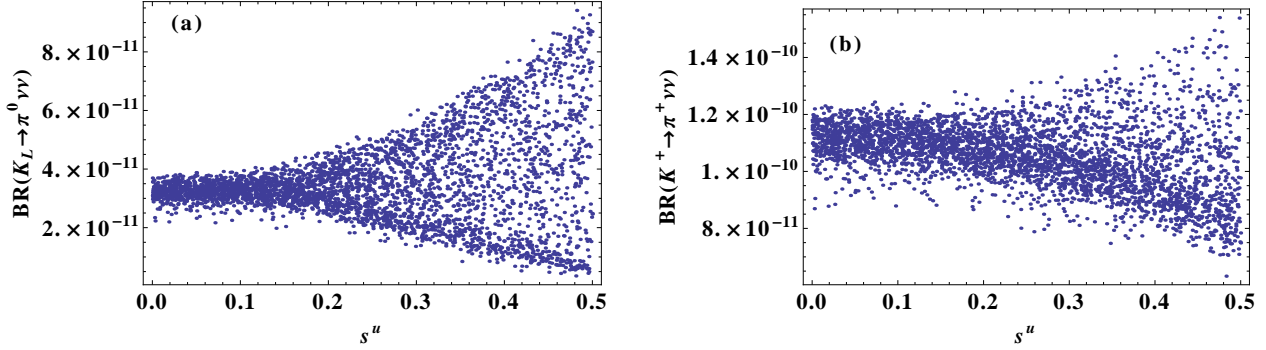


Figure 6: The predicted s^u dependence of (a) $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and (b) $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ at the SUSY scale of 50 TeV, where s^d is scanned in the region of $0 \sim 0.3$ independent of s^u .

To see the correlation between ϵ_K and the predicted $K_L \rightarrow \pi^0 \nu \bar{\nu}$ branching ratio, we show the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ versus the SUSY contribution of ϵ_K in Fig. 5, in which (a) $s^u = s^d = 0.3$ and (b) $s^u = 0.3$, $s^d = 0$. We do not find any correlation between them in the Fig. 5(a), where the gluino contribution to ϵ_K is still possible up to 10%. However, it is found a correlation between them as seen in Fig. 5(b), where the Z penguin mediated chargino dominates both SUSY contributions of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and ϵ_K since the down-squark mixing s_d vanishes with keeping $s_u = 0.3$. The chargino contribution to ϵ_K is at most 2%. This correlation is understandable from the difference of the phase structure between the penguin diagram and the box diagram of the chargino.

Thus, even if the SUSY scale is 50 TeV, ϵ_K could be deviated from the SM prediction in $\mathcal{O}(10\%)$ due to the gluino box diagram, whereas the chargino process deviates the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ from the SM prediction in the factor two.

Fig.6 shows the s^u dependence of $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ and $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ taking $s^u = 0 \sim 0.5$ in Fig.6 (a) and (b). We also scan s^d in the region of $0 \sim 0.3$ independent of s^u . In this plot, the SUSY contribution to ϵ_K is free ($0 - 40\%$), but the experimental constraint of ϵ_K with the error-bar of 90%C.L. is taken account. The predicted $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ could be large up to 8×10^{-11} , and $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ is up to 1.5×10^{-10} . Thus, the enhancement

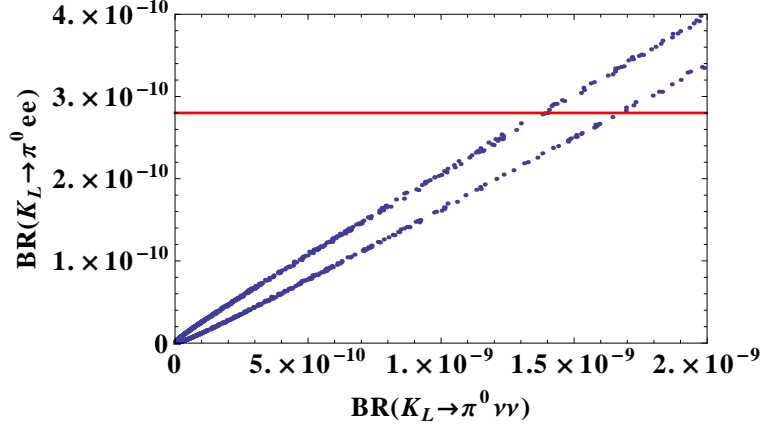


Figure 7: The predicted $BR(K_L \rightarrow \pi^0 e^+ e^-)$ versus $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ with $s^u = 0 \sim 0.3$ and $s^d = 0 \sim 0.3$ at the SUSY scale of 10 TeV. The red solid line denotes the upper-bound of the branching ratio $BR(K_L \rightarrow \pi^0 e^+ e^-)$.

from the SM prediction could be detectable even if the SUSY scale is 50TeV.

Before closing our numerical study, we would like to discuss correlations to other quantities which are sensitive to the NP. They are $K_L \rightarrow \pi^0 e^+ e^-$ process and the neutron electric dipole moment d_n . The $K_L \rightarrow \pi^0 e^+ e^-$ process is induced in similar way to $K_L \rightarrow \pi^0 \nu \bar{\nu}$. The distinguish feature of $K_L \rightarrow \pi^0 e^+ e^-$ mode is the contribution of the photon penguin. Moreover, one cannot neglect the long-distance effect from the photon exchange process [48]. Thus, the decay amplitude of $K_L \rightarrow \pi^0 e^+ e^-$ has both the short-distance effect and the long-distance effect, and the SM prediction of the branching ratio is around 3×10^{-11} , which is comparable to the SM prediction of $K_L \rightarrow \pi^0 \nu \bar{\nu}$. Since our interest here is to check whether the SUSY effect does not exceed the experimental bound of $K_L \rightarrow \pi^0 e^+ e^-$, we only consider the short distance contribution in our analysis. The experimental bound of the branching ratio $K_L \rightarrow \pi^0 e^+ e^-$ is $BR(K_L \rightarrow \pi^0 e^+ e^-)_{\text{exp}} < 2.8 \times 10^{-10}$ [43]. In the Fig.7, the predicted $BR(K_L \rightarrow \pi^0 e^+ e^-)$ vs. $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ plane are plotted with $s^u = 0 \sim 0.3$ and $s^d = 0 \sim 0.3$ at the 10TeV scale of the SUSY. There are two predicted lines in this figure. Because the decay amplitude $A(K_L \rightarrow \pi^0 e^+ e^-)$ is described by the sum of the SM and the SUSY contributions, there are two ways of taking the relative phase of \pm such as $A(K_L \rightarrow \pi^0 e^+ e^-) = A(K_L \rightarrow \pi^0 e^+ e^- : \text{SM}) \pm A(K_L \rightarrow \pi^0 e^+ e^- : \text{SUSY})$, which has two solutions giving the same absolute value of the decay amplitude. Then, we have two predicted values of $BR(K_L \rightarrow \pi^0 e^+ e^-)$ for the certain $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$. The both decay processes are dominated by the Z penguin mediated charginos, then, the branching ratios are determined by the final state couplings of $Z \nu \bar{\nu}$ and $Z e^+ e^-$, that is, the weak charges $Q_{ZL}^{(\nu)}$ and $Q_{ZL}^{(e)}$. Moreover, three flavors of neutrinos are summed for $K_L \rightarrow \pi^0 \nu \bar{\nu}$. Therefore, $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ is significantly larger than $BR(K_L \rightarrow \pi^0 e^+ e^-)$. On the other hand, in the SM, there are some contributions to $K_L \rightarrow \pi^0 e^+ e^-$ such as the photon exchange processes. So, $BR(K_L \rightarrow \pi^0 e^+ e^-)$ is comparable to $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ in the SM. In conclusion, the experimental upper bound of $BR(K_L \rightarrow \pi^0 e^+ e^-)$ excludes the region larger than $BR(K_L \rightarrow \pi^0 \nu \bar{\nu}) = 1.7 \times 10^{-9}$. However, if the long-distance effect is properly included [48], this

constraint becomes somewhat tight or loose depending on the relative sign between the SUSY contribution and the long-distance one.

The neutron electric dipole moment (EDM) d_n is well known as the sensitive probe for the NP, and so we have studied the correlation between the neutron EDM and the $K \rightarrow \pi^0 \nu \bar{\nu}$ branching ratio. It is found that our predicted $K \rightarrow \pi^0 \nu \bar{\nu}$ does not correlate with d_n . Suppose the SUSY contribution to the chromo-EDM of quarks through the gluon penguin mediated gluino [49]–[53], where the left-right mixing term of the down-squark is dominant. In our SUSY mass spectra, the left-right mixing is suppressed as discussed in section 3. Moreover, the CP violating phase dependence of d_n comes from the down-squark mixing matrix whereas the phase of $K \rightarrow \pi^0 \nu \bar{\nu}$ comes from the up-squark mixing matrix. Namely, those phase dependences are completely different each other. Therefore, we do not take account of the constraint from the experimental upper bound of the neutron EDM in our analyses.

5 Summary

We have studied the contribution of the high-scale SUSY to the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ processes by correlating with the CP violating parameter ϵ_K . These rare decays have important role of the decision of the CP phase in the CKM matrix, furthermore, they are also sensitive to the flavor structure of the NP.

Taking account of the recent LHC results for the Higgs discovery and the SUSY searches, we consider the high-scale SUSY at the 10 – 50 TeV scale. Then, we have discussed the SUSY effects to $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and ϵ_K in the framework of the mass eigenstate basis of the SUSY particles assuming the non-minimal squark (slepton) flavor mixing.

We have calculated the SUSY contribution to the branching ratios of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, where phase parameters are constrained by the observed ϵ_K . The Z penguin mediated chargino dominates the SUSY contribution for these decays. At the 10 TeV scale of the SUSY, its contribution can enhance the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in eight times compared with the SM predictions whereas the predicted branching ratio $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ increases up to three times of the SM prediction in the case of the up-squark mixing $s^u = 0.1$.

We have investigated the correlation between ϵ_K and the $K_L \rightarrow \pi^0 \nu \bar{\nu}$ branching ratio. Since the gluino box diagram dominates the SUSY contribution of ϵ_K up to 30%, there is no correlation between them. However, if the down-squark mixing is neglected compared with the up-squark mixing, the chargino process dominates both SUSY contributions of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ and ϵ_K . Then, it is found a correlation between them, but the chargino contribution to ϵ_K is at most 3%. It is concluded that ϵ_K could be deviated significantly from the SM prediction in $\mathcal{O}(10\%)$ due to the gluino box process, whereas the chargino process could enhance the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ in several times from the SM prediction.

Our predicted branching ratios depend on the mixing angle s^u significantly. The observed upper bound of $BR(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ gives the constraint for the predicted $BR(K_L \rightarrow \pi^0 \nu \bar{\nu})$ at s^u larger than 0.2.

Even if the SUSY scale is 50 TeV, the chargino process still enhances the branching ratio of $K_L \rightarrow \pi^0 \nu \bar{\nu}$ from the SM prediction in the factor two, and the ϵ_K is deviated from the SM prediction in $\mathcal{O}(10\%)$ unless the down-squark mixing s^d is suppressed.

We also discuss correlations to the $K_L \rightarrow \pi^0 e^+ e^-$ process and the neutron electric dipole moment d_n which are sensitive to the NP.

We expect the measurement of these processes will be improved by the J-PARC KOTO experiment and CERN NA62 experiment in the near future.

Acknowledgment

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Appendix A : SUSY Spectrum

In the framework of the MSSM, one obtains the SUSY particle spectrum which is consistent with the observed Higgs mass. The numerical analyses have been given in Refs. [54, 55]. At the SUSY breaking scale Λ , the quadratic terms in the MSSM potential is given as

$$V_2 = m_1^2 |H_1|^2 + m_2^2 |H_2|^2 + m_3^2 (H_1 \cdot H_2 + h.c.) . \quad (34)$$

The mass eigenvalues at the H_1 and $\tilde{H}_2 \equiv \epsilon H_2^*$ system are given

$$m_{\mp}^2 = \frac{m_1^2 + m_2^2}{2} \mp \sqrt{\left(\frac{m_1^2 - m_2^2}{2}\right)^2 + m_3^4} . \quad (35)$$

Suppose that the MSSM matches with the SM at the SUSY mass scale $Q_0 \equiv m_0$. Then, the smaller one m_-^2 is identified to be the mass squared of the SM Higgs H with the tachyonic mass. The larger one m_+^2 is the mass squared of the orthogonal combination \mathcal{H} , which is decoupled from the SM at Q_0 , that is, $m_{\mathcal{H}} \simeq Q_0$. Therefore, we have

$$m_-^2 = -m^2(Q_0) , \quad m_+^2 = m_{\mathcal{H}}^2(Q_0) = m_1^2 + m_2^2 + m^2 , \quad (36)$$

with

$$m_3^4 = (m_1^2 + m^2)(m_2^2 + m^2) , \quad (37)$$

which leads to the mixing angle between H_1 and \tilde{H}_2 , β as follows:

$$\tan^2 \beta = \frac{m_1^2 + m^2}{m_2^2 + m^2} , \quad H = \cos \beta H_1 + \sin \beta \tilde{H}_2 , \quad \mathcal{H} = -\sin \beta H_1 + \cos \beta \tilde{H}_2 . \quad (38)$$

Thus, the Higgs mass parameter m^2 is expressed in terms of m_1^2 , m_2^2 and $\tan \beta$:

$$m^2 = \frac{m_1^2 - m_2^2 \tan^2 \beta}{\tan^2 \beta - 1} . \quad (39)$$

Below the Q_0 scale, in which the SM emerges, the scalar potential is the SM one as follows:

$$V_{SM} = -m^2 |H|^2 + \frac{\lambda}{2} |H|^4 . \quad (40)$$

Here, the Higgs coupling λ is given in terms of the SUSY parameters at the leading order as

$$\lambda(Q_0) = \frac{1}{4}(g^2 + g'^2) \cos^2 2\beta + \frac{3h_t^2}{8\pi^2} X_t^2 \left(1 - \frac{X_t^2}{12}\right) , \quad X_t = \frac{A_t(Q_0) - \mu(Q_0) \cot \beta}{Q_0} , \quad (41)$$

and h_t is the top Yukawa coupling of the SM. The parameters m_2 and λ run with the SM Renormalization Group Equation down to the electroweak scale $Q_{EW} = m_H$, and then give

$$m_H^2 = 2m^2(m_H) = \lambda(m_H) v^2 . \quad (42)$$

It is easily seen that the VEV of Higgs, $\langle H \rangle$ is v , and $\langle \mathcal{H} \rangle = 0$, taking account of $\langle H_1 \rangle = v \cos \beta$ and $\langle H_2 \rangle = v \sin \beta$, where $v = 246 \text{ GeV}$.

Let us fix $m_H = 126 \text{ GeV}$, which gives $\lambda(Q_0)$ and $m^2(Q_0)$. This experimental input constrains the SUSY mass spectrum of the MSSM. We consider the some universal soft breaking parameters at the SUSY breaking scale Λ as follows:

$$\begin{aligned} m_{\tilde{Q}_i}(\Lambda) &= m_{\tilde{U}_i^c}(\Lambda) = m_{\tilde{D}_i^c}(\Lambda) = m_{\tilde{L}_i}(\Lambda) = m_{\tilde{E}_i^c}(\Lambda) = m_0^2 \quad (i = 1, 2, 3) , \\ M_1(\Lambda) &= M_2(\Lambda) = M_3(\Lambda) = m_{1/2} , \quad m_1^2(\Lambda) = m_2^2(\Lambda) = m_0^2 , \\ A_U(\Lambda) &= A_0 y_U(\Lambda) , \quad A_D(\Lambda) = A_0 y_D(\Lambda) , \quad A_E(\Lambda) = A_0 y_E(\Lambda) . \end{aligned} \quad (43)$$

Therefore, there is no flavor mixing at Λ in the MSSM. However, in order to consider the non-minimal flavor mixing framework, we allow the off diagonal components of the squark mass matrices at the 10% level, which leads to the flavor mixing of order 0.1. We take these flavor mixing angles as free parameters at low energies.

Now, we have the SUSY five parameters, Λ , $\tan \beta$, m_0 , $m_{1/2}$, A_0 , where $Q_0 = m_0$. In addition to these parameters, we take $\mu = Q_0$. By fixing Λ , Q_0 and $\tan \beta$, we tune $m_{1/2}$ and A_0 in order to obtain $m^2(Q_0)$ and $\lambda_H(Q_0)$ which realize the correct electroweak vacuum with $m_H = 126 \text{ GeV}$. Then, we obtain the SUSY particle spectrum. We consider the two case of $Q_0 = 10 \text{ TeV}$ and 50 TeV . The input parameter set and the obtained SUSY mass spectra at Q_0 are summarized in Table 1, where we use $\overline{m}_t(m_t) = 163.5 \pm 2 \text{ GeV}$ [43, 56].

Input at Λ and Q_0	Output at Q_0
at $\Lambda = 10^{17} \text{ GeV}$, $m_0 = 10 \text{ TeV}$, $m_{1/2} = 6.2 \text{ TeV}$, $A_0 = 25.803 \text{ TeV}$; at $Q_0 = 10 \text{ TeV}$, $\mu = 10 \text{ TeV}$, $\tan \beta = 10$	$m_{\tilde{g}} = 12.8 \text{ TeV}$, $m_{\tilde{W}} = 5.2 \text{ TeV}$, $m_{\tilde{B}} = 2.9 \text{ TeV}$ $m_{\tilde{b}_L} = m_{\tilde{t}_L} = 12.2 \text{ TeV}$ $m_{\tilde{b}_R} = 14.1 \text{ TeV}$, $m_{\tilde{t}_R} = 8.4 \text{ TeV}$ $m_{\tilde{s}_L, \tilde{d}_L} = m_{\tilde{c}_L, \tilde{u}_L} = 15.1 \text{ TeV}$ $m_{\tilde{s}_R, \tilde{d}_R} \simeq m_{\tilde{c}_R, \tilde{u}_R} = 14.6 \text{ TeV}$, $m_{\mathcal{H}} = 13.7 \text{ TeV}$ $m_{\tilde{\tau}_L} = m_{\tilde{\nu}_{\tau L}} = 10.4 \text{ TeV}$, $m_{\tilde{\tau}_R} = 9.3 \text{ TeV}$ $m_{\tilde{\mu}_L, \tilde{e}_L} = m_{\tilde{\nu}_{\mu L}, \tilde{\nu}_{e L}} = 10.8 \text{ TeV}$, $m_{\tilde{\mu}_R, \tilde{e}_R} = 10.3 \text{ TeV}$ $X_t = -0.22$, $\lambda_H = 0.126$ $m_1^2 = 1.84857 \times 10^8 \text{ GeV}^2$, $m_2^2 = 1.83996 \times 10^6 \text{ GeV}^2$, $m^2 = 8691 \text{ GeV}^2$
at $\Lambda = 10^{16} \text{ GeV}$, $m_0 = 50 \text{ TeV}$, $m_{1/2} = 63.5 \text{ TeV}$, $A_0 = 109.993 \text{ TeV}$; at $Q_0 = 50 \text{ TeV}$, $\mu = 50 \text{ TeV}$, $\tan \beta = 4$	$m_{\tilde{g}} = 115.6 \text{ TeV}$, $m_{\tilde{W}} = 55.4 \text{ TeV}$, $m_{\tilde{B}} = 33.45 \text{ TeV}$ $m_{\tilde{b}_L} = m_{\tilde{t}_L} = 100.9 \text{ TeV}$ $m_{\tilde{b}_R} = 104.0 \text{ TeV}$, $m_{\tilde{t}_R} = 83.2 \text{ TeV}$ $m_{\tilde{s}_L, \tilde{d}_L} = m_{\tilde{c}_L, \tilde{u}_L} = 110.7 \text{ TeV}$, $m_{\tilde{s}_R, \tilde{d}_R} = 110.7 \text{ TeV}$ $m_{\tilde{c}_R, \tilde{u}_R} = 105.0 \text{ TeV}$, $m_{\mathcal{H}} = 83.1 \text{ TeV}$ $m_{\tilde{\tau}_L} = m_{\tilde{\nu}_{\tau L}} = 63.6 \text{ TeV}$, $m_{\tilde{\tau}_R} = 54.6 \text{ TeV}$ $m_{\tilde{\mu}_L, \tilde{e}_L} = m_{\tilde{\nu}_{\mu L}, \tilde{\nu}_{e L}} = 63.8 \text{ TeV}$, $m_{\tilde{\mu}_R, \tilde{e}_R} = 55.0 \text{ TeV}$ $X_t = -0.65$, $\lambda_H = 0.1007$ $m_1^2 = 6.49990 \times 10^9 \text{ GeV}^2$, $m_2^2 = 4.06235 \times 10^8 \text{ GeV}^2$, $m^2 = 8840 \text{ GeV}^2$

Table 1: Input parameters at Λ and the obtained SUSY spectra at $Q_0 = 10$ and 50 TeV .

Appendix B : Z penguin amplitude mediated charginos

We present the expression for the Z penguin amplitude mediated the chargino, $P_{\text{ZL}}^{sd}(\chi^\pm)$ in our basis [44] as follows:

$$P_{\text{ZL}}^{sd}(\chi^\pm) = \frac{g_2^2}{4m_W^2} \sum_{\alpha,\beta,I,J} (\Gamma_{CL}^{(d)\dagger})_I^{\alpha d} (\Gamma_{CL}^{(d)})_J^{\beta s} \left\{ \delta_I^J (U_+^\dagger)_\beta^1 (U_+)_1^\alpha [\log x_I^{\mu_0} + f_2(x_\alpha^I, x_\beta^I)] \right. \quad (44)$$

$$\left. - 2\delta_I^J (U_-^\dagger)_\beta^1 (U_-)_1^\alpha \sqrt{x_\alpha^I x_\beta^I} f_1(x_\alpha^I, x_\beta^I) - \delta_\beta^\alpha \left(\tilde{\Gamma}_L^{(u)} \right)_I^J f_2(x_I^\alpha, x_J^\alpha) \right\} , \quad (45)$$

where

$$(\Gamma_{CL}^{(d)})_I^{\alpha q} \equiv (\Gamma_L^{(u)} V_{\text{CKM}})_I^q (U_+)_1^\alpha + \frac{1}{g_2} (\Gamma_R^{(u)} \hat{f}_U V_{\text{CKM}})_I^q (U_+)_2^\alpha , \quad (46)$$

and

$$\left(\tilde{\Gamma}_L^{(u)} \right)_I^J \equiv \left(\Gamma_L^{(u)} \Gamma_L^{(u)\dagger} \right)_I^J , \quad (47)$$

with $q = s, d$, $I = 1 - 6$ for up-squarks, and $\alpha = 1, 2$ for charginos. The V_{CKM} is the CKM matrix, and U_\pm are the 2×2 unitary matrices which diagonalize the chargino mass matrix M_C :

$$U_-^\dagger M_C U_+ = -\text{diag} M_C^\alpha , \quad (\alpha = 1, 2) . \quad (48)$$

The \hat{f}_U denotes the yukawa coupling defined by $\hat{f}_U v \sin \beta = \text{diag}(m_u, m_c, m_t)$. The loop integral functions are given as:

$$f_n(x, y) = \frac{1}{x - y} \left(\frac{x^n \log x}{x - 1} - \frac{y^n \log y}{y - 1} \right) , \quad (49)$$

with

$$x_\alpha^I = \frac{m_{\chi_\alpha}^2}{\tilde{m}_I^2} , \quad x_I^{\mu_0} = \frac{\tilde{m}_I^2}{\mu_0^2} , \quad (50)$$

where $\mu_0 = Q_0$ is taken in our framework.

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